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**DUE 04 SEP 2019**

**AAE 537 Homework #1**

**Fall, 2019**

1. Using the standard atmosphere model provided, compute conditions along constant q trajectories of q=800, 1000, and 1200 psf. Produce the following plots:
   1. The flight corridor relating Mach number (x-axis) to altitude (y-axis) for each q (on the same plot)
   2. Stagnation temperature vs altitude for the three q values noted
   3. Recovery temperature vs. altitude for the three q values noted
   4. Stagnation pressure vs. altitude for the three q values noted
   5. Static pressure vs. altitude for the three q values noted

**ANSWER 1a:**

Flying at a certain altitude and a constant psf will result in only one Mach number, since Mach is dependent upon the temperature of that altitude, and psf is dependent on the pressure of that altitude.

Using the MATLAB script “atmosphere.m”, I was able to build a matrix of altitude parameters from 0-160,000 ft that included: [temp,press,rho,Hgeopvector] where Hgeopvector in my script became the geometric altitude (constant gravity) instead of geopotential altitude.

With that script, I then calculated the Mach number at each altitude for a given dynamic pressure. With a constant dynamic pressure, then for instance at 0 feet I had a set temperature and pressure and could output the Mach number for that altitude. Running the script across all altitudes resulted in Mach numbers indexed to the altitude vector for the three given dynamic pressures.

I then plotted Mach to altitude with a vertical line indicating orbital velocity. (See **Figure 1a**).This graph tells me that to achieve hypersonic flight beyond Mach 5, I need to be above 80,000 feet, else the structural load limit will be rapidly exceeded.

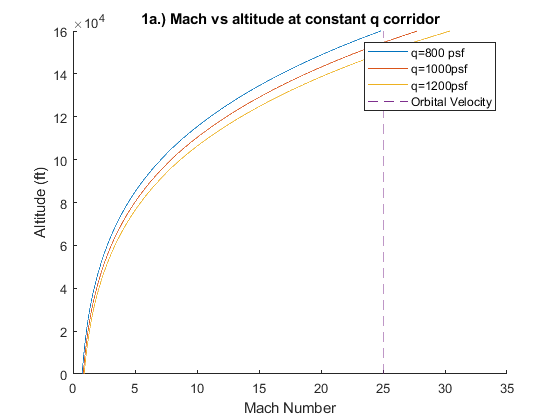


Figure 1a

**ANSWER 1b:**

Stagnation temperature (also total temperature) is the static temperature plus all the dynamic energy contained within, as if isentropically brought to a point of stagnation on the nose or leading edge of a wing.

The Recovery Temperature Ratio is defined as the ratio of the freestream static and stagnation values. Therefore, the Recovery Temperature Ratio is a function of only Mach number, and is isentropically related to the Recovery Pressure Ratio :

is freestream static temperature from “atmosphere.m”

is freestream Mach number derived from part 1a (varies w/ altitude and dynamic pressure)

is assumed to be 1.4 throughout this homework (value lowers above 600C)

Mach number was indexed to altitude, creating a vector of the same size. For instance, the tenth element of altitude (900’) was indexed to the tenth element of Mach (q=800psf at 900’ is M=0.747) and the tenth element of freestream temperature ( at 900’ = 515R).

Because we’re flying at a constant q, whenever I plot against Mach I’m technically plotting against altitude. I then simply call the same number in the altitude vector that I did for Mach. In this way I had created a total temperature vector that was indexed to altitude. I then plotted these three indexed stagnation temperature vectors against the original altitude vector (See **Figure 1b**).

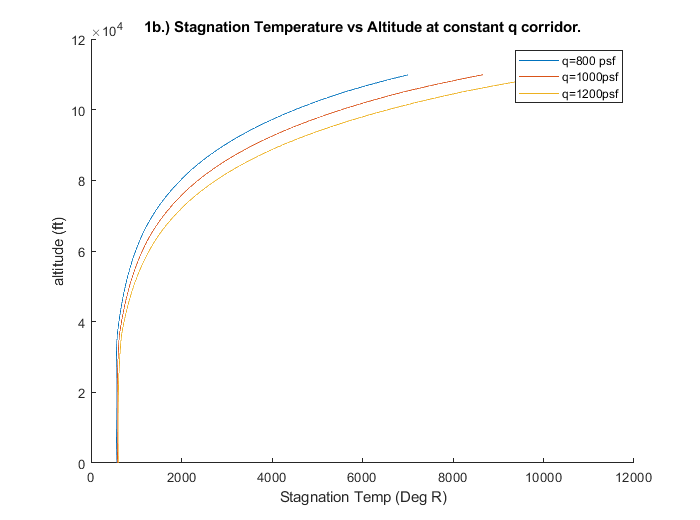


Figure 1b

**Figure 1b** (above) shows that temperature is constant for the constant q until past 30,000 feet. It then rapidly gets uncontrollable if staying in the same dynamic corridor. At about 60,000’ the air rapidly thins and the need to accelerate Mach suddenly increases (as seen with **Figure 1a**). The temperature increases with the square of Mach, so even with cooler temperatures at altitude, the Mach number drives significant heating to the aircraft beginning near Mach 3. At the far right of the graph are temperatures that nearly no substance can take, which is what drives the need for ablative shields on hypersonic spacecraft re-entering the atmosphere.

**ANSWER 1c:**

Recovery temperature is the static temperature at the bottom of the boundary layer in a Mach environment. Assuming perfect flow, all the kinetic energy (KE) of the freestream would be slowed to a stop on the surface and converted to heat energy. However, some of the KE is converted to “viscous dissipation” where pressure changes and current eddies absorb some energy. The result is slightly lower temperature at the surface, and a true measure for maximum possible skin temperature of the aircraft.

Earlier, it was stated that total temperature is a recovery from static temperature.

Recovery temperature (not the ratio) is close to the above equation but with a correction factor:

Here, and Pr is Prandtl’s Number ( )

Prandtl’s number is far from constant; it varies with temperature, pressure, and substance. Since the air is a uniform mixture, it would vary with altitude with a range from about 0.7-1.0 depending on how high one looks. I looked up the standard sea level value of Prandtl’s number for dry air and found . Since this homework is simply an exercise on relations, I’ll use this number throughout the atmosphere.

From here I can tell that the recovery temperature graph will look very similar to total temperature, but with slightly smaller values. Recovery Temperature indexed to Mach does necessarily relate it to altitude vector of the same size, and is plotted in **Figure 1c**.

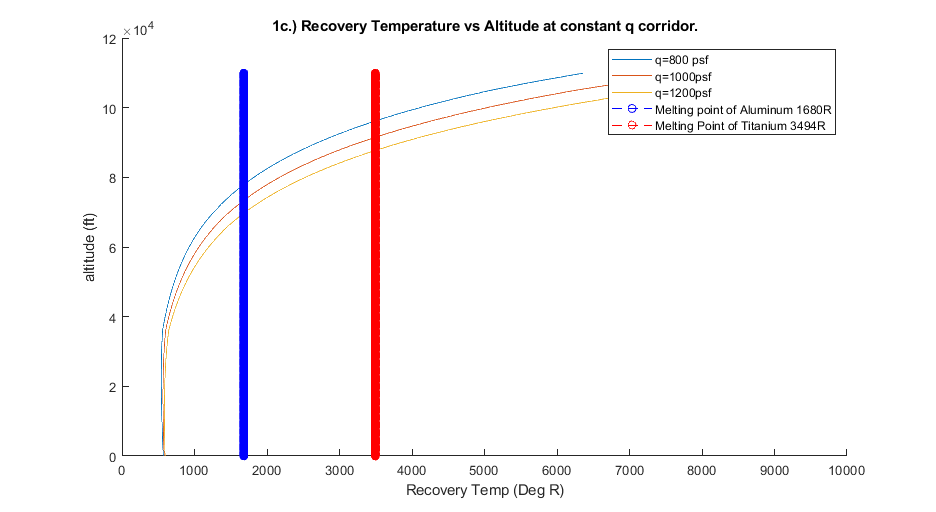


Figure 1c

Here in the above figure, one can see that above 60,000 feet (when Mach is above 3) the temperature skyrockets to the right along the x-axis just like with the total temperature as predicted. Because recovery temperature is the temperature that reaches the skin, it’s useful to put that into context. Here I drew two lines indicating the melting point of two common aerospace metals Al and Ti. However, these are the melting points; strength loss occurs well below the melting point. Here it’s clearly obvious that Aluminum could never work for Mach 3 flight around 80,000 feet.

**ANSWER 1d**

Stagnation pressure vs. Altitude is calculated much the same way that stagnation temperature was.

With high speed flight, even in thin upper atmosphere, I expect that pressures will grow exponentially. The results are plotted below in **Figure 1d**.

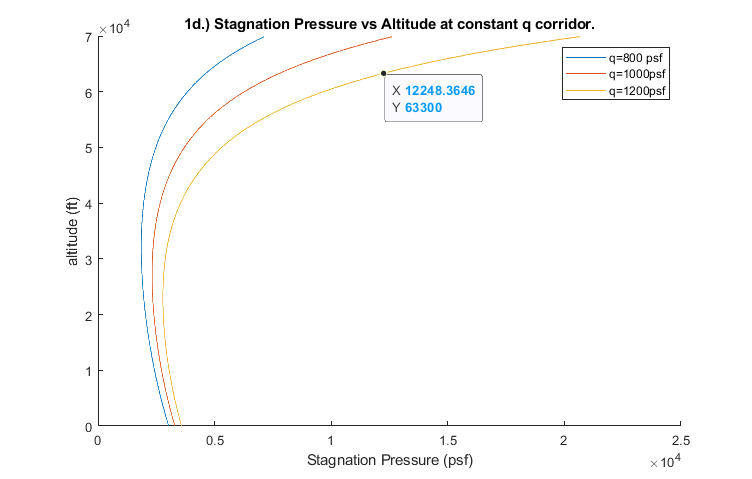


Figure 1d

Surprisingly (to me), the total pressure initially drops with an increase in altitude before beginning to rise again around 40,000’ when flying a constant dynamic pressure corridor. This graph is significantly zoomed in compared to the other graphs because the pressure grows wildly out of control otherwise. The ‘1’ on the x-axis corresponds to a stagnation pressure of 10,000 psf.

**ANSWER 1E**

Static pressure is, I expect, more nuanced than simply static atmospheric pressure from the altitude model. Recovery Pressure Ratio defined as the ratio of total pressure to static pressure:

I’ve already plotted total pressure, and from that definition static won’t vary with dynamic pressure. However, the same equation appears in other places when relating two static pressures across a certain boundary, such as . The equations are all isentropic. So, I’m going to assume that static pressure is isentropically related to recovery temperature.

This new “recovery pressure” is calculated in this manner:

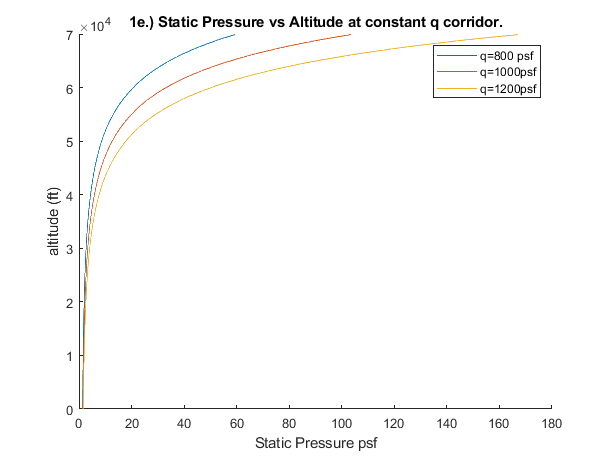
It’s very similar to total pressure () but with the added Prandtl factor. Recovery pressure (Static Pressure) is plotted below in **Figure 1e**. At low Mach numbers, the static pressure is very near zero until about 50,000 feet (~ Mach 2.2 for all corridors). Mach compression then causes the static pressure to run away just like temperature.

Figure 1e

1. In this problem, develop Isp-Mach relationships for the following ideal systems:  
     
   i)  Turbojet engine with CPR=10  
   ii) Afterburning Turbojet with CPR=10  
   ii) Turbofan engine with CPR=30, FPR=2, BPR=6  
   iii) Ramjet engine with start condition at M=1.7  
   iv) Scramjet engine with start condition at M=3.5  
   v) Bipropellant rocket engine with liquid oxygen oxidizer at Pc=3000 psi  
     
   Assume that all systems are operating within an airbreathing corridor at a dynamic pressure of 1500 psf.  For all airbreathing systems:  
    -  Consider kerosene fuel with a heating value (heat of combustion) of 18,500 BTU/lbm.  
    -  Assume ideal components throughout with the exception of the inlet. For this component assume inlet recovery per Mil Std 5008B.  
    -  Assume an average Cp of 0.3 BTU/lbm-deg F.  
     
   Turbine inlet temperatures are limited to 2600 F, ramjet burner or afterburner exit temperature is limited to 3500 F and scramjet burner exit is limited to 4000 F.  You may assume that all nozzles are perfectly expanded at all Mach conditions.  The rocket engine is operating at optimal O/F. Take the following steps in your study:  
     
   a)  Derive relationships for specific thrust and Isp for the turbofan engine.  Here, FPR and BPR are fan pressure ratio and bypass ratio respectively.  
   b)  Run CEA at a number of altitudes to determine rocket performance as a function of Mach number  
   c)  Generate Isp-Mach relationships for all systems and plot all results on the same graph.  
   d)  Discuss the implications of your results, i.e. which system is preferred over which Mach range?  Why?

**GIVENS**

(Chamber pressure)

Kerosene Fuel in Rocket assumed to be RP-1 (a kerosene derivative)

Turbojet, Afterburning turbojet, Turbofan, and Rocket start from rest

Ramjet starts at M=1.7

Scramjet starts at M=3.5

Mil Std 5008B for inlet recovery between stations 0 and 2 (freestream to first compressor face),

This standard isn’t ever experimentally accurate but is a good starting point for inlet design.

:

:

:

Everything else is ideal and perfect.

a)  Derive relationships for specific thrust and Isp for the turbofan engine.  Here, FPR and BPR are fan pressure ratio and bypass ratio respectively.

Notable that BPR is a ratio of mass flows, not pressure.

First things first, assume turbofan has no afterburner (not always true).

It’s asking for me to derive the Specific Thrust relationship, so I’m going to assume derive the equation for specific thrust. Isp is the same thing but divided by gravity.

I’m going to use the same station numbers as the turbojet, but add a station for the fan in front of the turbojet compressor. The fan isn’t in the freestream; the engine nacelle lip is.

1. freestream
2. nacelle lip (diffuser/inlet/intake begin, designed with Mil Std 5008B)

19 turbofan inlet (end of diffuser)

2

2 compressor inlet / turbofan exhaust

3 compressor exit / burner inlet

4 burner exit / turbine inlet (Highest temp?)

5 turbine exit

6 afterburner

7 nozzle inlet

8 nozzle throat

9 nozzle exit (core exhaust)

9\* turbofan exhaust

These numbers will be used consistently here forward.

From the notes, the equation for specific thrust (turbojet, no A/B) is:

I don’t have the exit velocity, but I do have enough information to calculate the temperature and pressure ratios across the engine, which will give a velocity ratio. **I’ll have to re-derive the above equation for a turbofan engine.** Like the examples, I’ll start from the front and step through the back until I obtain enough ratios for the specific thrust of a turbofan.

*FREESTREAM*

From “atmosphere.m” I have static temperature and pressure at all altitudes.

*INLET*

Ideal engine, ideal conditions, I can calculate stagnation values using recovery ratios:

*ACROSS DIFFUSER*

Keeping standard stations as indicated above, the intake end is station 19 for the turbofan face.

Mil Std 5008B:

:

:

*ACROSS FAN*

*ACROSS COMPRESSOR*

*ACROSS BURNER*

Ideal, but not isentropic. Assume nearly constant low Mach number, so Rayleigh flow simplifies:

*ACROSS TURBINE*

Modelled the same as the compressor. Adiabatic but with flow work (shaft work), so the ratios aren’t unity but are instead isentropically related. The turbine drives both the compressor and the turbofan with no losses. The amount of work extracted determines the total temperature at the back end of the turbine, so a work balance is necessary.

Define fuel flow:

This says that, if maintaining constant Tt4, then as speed increases and Tt3 increases, fuel flow will decrease. With decreasing fuel flow and a smaller temp difference across the engine, thrust will also go down with higher speed (in this idealization).

Rewrite fan mass flow with bypass ratio, rewrite turbine mass flow with fuel flow:

Assume constant heat coefficients, steady mass flow

Solving for Tt5,

(Without a turbofan, simply delete the third term)

*NOZZLE*

Turbine ends at station 5 and Nozzle starts at station 7. Without an afterburner, assume Stations 5 and 7 are identical.

Ideal nozzle is an adiabatic process with no heat transfer and no shaft work. Also assume no Mach losses, so isentropic and the pressure/temperature ratios are unity.

*CALCULATIONS FOR CORE THRUST*

Stagnation to static temperature ratio at nozzle exit:

**1.)**

Note the diffuser is not unity, because above Mach 1 it has a different relation (programmed as piece-wise function IAW Mil Std 5008B).

Nozzle is isentropic, so

Perfect expansion, low Mach burner, and isentropic nozzle all give unity pressure ratios.

**2.)**

Equate the two grayed equations 1.) and 2.),

Which yields a new definition:

This means the net energy addition in the entire ideal cycle (turbojet or turbofan) occurs in the burner section. Everything else is adiabatic. If I were deriving an afterburner, I’d find that there would be two energy additions and multiply in the A/B ratio. Based on this, I can expect that the net energy addition in the fan will be one.

From recovery temperature ratio,

I can infer that

From equation 2.),

Relating inlet and exit Mach from the found derivations,

(In subsonic flight without a turbofan, the diffuser and fan tau’s are unity)

Now with both Mach and Temperature ratios across the engine, I can calculate the velocity ratio, because change in temperature is related to change in U which was converted to KE.

**3.)**

I have now found the velocity ratio from the jet exhaust to the freestream, with this I could calculate the turbojet thrust from just the core.

CALCULATIONS FOR FAN THRUST

At the beginning of this problem I wrote:

For the entire turbofan,

Here, is the exhaust speed of the fan. I’m not going to combine the exhaust flows for this homework assignment but instead keep them separate.

Finding the velocity ratios across the fan, and again assuming the fan is perfectly expanded on the back end (all nozzles are perfect here)

Ambient static pressure = turbofan exhaust static pressure = core exhaust static pressure

Where 9\* denotes turbofan exhaust (not mixing with core)

The fan exhaust is isentropic, so

Equating the two,

There’s no net energy addition in the fan, and the work extracted is accounted for with the core equations.

Solving for Mach at turbofan exhaust,

Recalling M0,

Dividing for Mach ratio,

As above, multiply by temperature ratio to get velocity. Except here, temperature ratio was found to be unity:

**4.)**

Recalling equation 3.)

I now have enough information to solve for specific thrust for the entire engine.

I need the mass flow specific for all air moving through the engine:

This is valid for a perfectly expanded nozzle and fuel flow rates much less than one. Substituting the velocity ratio I just derived,

Through the derivation I was able to define all the above parameters:

Turbofan Specific Thrust = Mass-of-air specific thrust, in ft/s

Thrust per mass of air moving through engine

Turbofan Specific Impulse = Weight-of-fuel specific thrust

With all the above information, I’ve finally answered question 2a:

Derived Relationship for Specific Thrust for Turbofan:

Derived Relationship for ISP for Turbofan:

2b) Run CEA at a number of altitudes to determine rocket performance as a function of Mach number

Problem Type ROCKET

Pressures Worst case, 1 atm

Fuel RP-1 (highly-refined kerosene)

Oxidizer O2(L)

Oxidizer/Fuel ratio o/f: Weight ratio, 2.56 (from Wikipedia)

Exit conditions: Pc/Pe (3000 psi / static pressure at altitude, evenly spaced along

q corridor indexed to Mach 7)

[1.42 2.02 2.97 4.48 7.02 11.09 17.59 27.77 43.88 68.61]

Final Equilibrium

Isp, M/SEC (I specified English units in, assuming it converted correctly to m/s)

Output after running 10 times at each of above exit conditions:

[ 893.4, 1253.9, 1545.4, 1795.9, 2025.4, 2226.5, 2404.8, 2562.4, 2704.8, 2831.4 ]

Divide by 9.8 m/s^2 to get seconds, then plot Isp against the Mach from which pressure was derived.

Relatively constant ISP that decreases slightly with increasing Mach at same altitude because of pressure drag. With Mach corridor, altitude increases reducing static pressure

2c) Generate ISP-Mach relationships for all systems and plot all results on the same graph

In the same way that the ISP-Mach relationship for ISP was built, similar relations were derived (albeit more rapidly) inside the MATLAB code attached.

Restrictive assumptions **(No Mach given, per Dr. Heister email treat scramjet as ramjet) but with higher Scramjet temps:**

A close up of a map

Description automatically generated

**2d) Discuss the implications of your results, i.e. which system is preferred over which Mach range? Why?**

Q=1500 psf means initial mach is supersonic. A turbofan with a BPR of 6 (commercial jet engine) wouldn’t really run well supersonic, but we’re ignoring Mach losses and discussing ISP at high level overview.

An ISP of 300 seconds means that an engine producing 3000 lbs of thrust requires 10 lbs of onboard propellant per second. A higher Isp system would require less fuel for the same amount of thrust. Doubling the Isp to 600 seconds for the same thrust output of 3000 lbs would burn only 5 lbs per second. That’s Isp, or more simply, it’s how much bang for the buck? Note that it’s “onboard” propellant, so for airbreathing engines, the mass-flow of air through the engine is not considered. The air isn’t onboard. For rockets, the oxidizer is carried onboard, and so the oxygen is counted as part of the Isp.

A turbofan engine gets the most bang for the buck at low speeds, because with small increases in fuel, you get huge increases in mass flow. Thrust, from the above equations, either need low mass experiencing a high delta-v (like the turbojet core) or a high mass experiencing a low delta-v (like the fan bypass). Because a small amount of extra fuel provides energy to so much more air mass than a regular turbojet, the turbofan can produce more thrust per pound of propellant. Therefore it has the highest Isp making it the most efficient at low speeds. This is why airliners prefer turbofans over turbojets. Also, with blending of the fan/jet exhaust, the combined exit velocity is lower, which reduces noise signature.

However, the large frontal area of the turbofan is susceptible to high ram drag. At high mach numbers, the ram drag dominates, and Isp drops off. This is where the turbojet comes in. With no bypass, it doesn’t have the above efficiency advantages and has a lower Isp. However, it can run at higher speeds with lower ram drag and could produce more thrust at the same speed (but with more fuel burned). Turbojets are found on many military aircraft, including the one I flew in the Marines (Pratt & Whitney J-52 non A/B)

The turbojet also hits a limit where ram drag, Mach effects, and compression raise the burner inlet temperature to a point there the burner exit (turbine inlet temperature) could be raised beyond limits. To compensate, fuel flow to the burner is reduced until no more thrust is being produced. To counteract, an afterburner could be added. The burner section isn’t stochiometric, and so a lot of oxygen remains coming out of the turbine. More fuel is added to the oxygen-rich exhaust, and the combustion process drastically increases the exit velocity of the exhaust, which increases thrust dramatically (and maximum speed) without exceeding the turbine inlet temperature. This is found on fighters that need high thrust at all speeds, and for fighters that want super-cruise. It’s terribly fuel-inefficient, though.

But there comes a point when turbomachinery can’t keep up with Mach losses, even with specially designed intakes. At such high speeds, mechanical compression isn’t even necessary and a properly designed intake can perform as well or better than turbomachinery for compression. No compressor means no turbine, and much less weight (with a downside of unable to ignite on deck). The ramjet’s limit is then only a temperature limit, and the burner is much more efficient since all the heat of combustion is going to the exhaust velocity instead of being absorbed as work by the turbine and compressor. A higher exit velocity means more thrust. However, the ramjet burning without machinery means it’s essentially an afterburner in a tube, and so for the same thrust it burns more fuel than a comparable turbojet. Speaking of afterburner, that’s basically how the J-58 on the SR-71 functioned: dumping excess compressor air into the afterburner to function as a ramjet. The ramjet (afterburner) produced 83% of the thrust while the turbomachinery only produced 17%. Naturally, so much fuel at high speeds reduces the Isp, and so we see ramjet curve below the turbojet curve. Turbojets likely wouldn’t operate at Mach 5+ like the graph indicates, but here we’re not accounting for Mach losses in the inlet. Again, just an idealization for an overview.

At higher and higher Mach, a ramjet becomes inefficient due to Mach losses. A ramjet is subsonic combustion, but at hypersonic speeds it becomes inevitable that shocks are present in the burner, implying supersonic combustion. This is where the scramjet comes in. For the purposes of this homework, the scramjet is modeled exactly the same as the ramjet, except that it has a higher temperature tolerance. Because of idealized assumptions, the graph shows the scramjet performing much the same as the ramjet, but with a slightly higher final Mach number (0.03 higher).

In real life, Rayleigh flow would dictate scramjet burner patterns, and the subsonic-burning ramjet would have its performance drop off much, much sooner. The advantage of the scramjet would be more obvious when accounting for Mach losses.

A rocket is the least efficient with the lowest Isp. Consider the Saturn V, which burned RP-1. It consumed approximately 11,000 lbs of fuel before liftoff while the engines got up to speed. Millions of pounds of fuel burned appears wasteful, but a rocket is used for two things: a need for high acceleration (such as missiles), and a need to operate hypersonically or in a vacuum, like with spaceships. After the point where hypersonic engines no longer function either due to temperature limits or due to altitude, a rocket becomes necessary. For now, space vehicles just use the rocket from rest instead of utilizing the other higher impulse systems due to difficulties in designing combined cycle engines. The high thrust found on space vehicles also drastically outperforms virtually every air breathing system available. But getting small payloads to orbit may one day use combined cycle, and would incorporate one of the higher Isp systems on the graph before eventually converting to a rocket at higher speed/altitude. Rocket thrust is constant, and Isp on a rocket is nearly constant with Mach, except for pressure drag reducing with altitude thereby increasing net thrust which give an increase in net Isp.

1. **Write down 10 things you learned from Kelly Johnson’s article on the SR-71.**

1. It was originally called RS-71 for Reconnaissance-Strike, but when the President erroneously announced it as SR-71, the program managers decided to adjust the name rather than correct him (likely to project an image of cohesion rather than confusion?) SR became Strategic Reconnaissance, which in my opinion fits better anyways.

2. In full ramjet mode above Mach 3, the turbojet core is only producing 17% of the thrust. That’s beyond unbelievable, and it’s more of a supercharged afterburner than anything else. The air is bypassing the core via six wide ducts (I saw in person at Udvar-Hazy) and I thought there would be significant losses piping the air through those curved pipes. Producing 83% of the thrust from these pipes at Mach 3 is unbelievable.

3. Everything had to be invented, including new wires since insulation from the skin’s heat only goes so far.

4. Hydraulic fluid had to be invented; the only one that worked at high speeds was powdered and couldn’t work at takeoff.

5. New fuel had to be invented. I knew that the fuel cooled the jet skin (and that it necessarily slows down when at low fuel), but I didn’t know the hydrocarbons could break down in doing so.

6. I knew the fuel cooled the skin but didn’t know it also cooled the tires. The wheels retracted into the middle of the tanks. Smart!

7. Pursuing hydrogen as a fuel was another thing I learned. In-flight refueling is already hazardous, and hydrogen even moreso. But getting tankers like today’s KC-135’s to carry hydrogen then penetrate other borders would either require lying about the fuel on board, or giving away the fact that the SR-71 is coming there to refuel and potentially compromising missions. I didn’t know hydrogen was being pursued, but politically and strategically it made sense to instead go with different hydrocarbons.

8. Speaking of hydrogen, I didn’t even think that it could only be stored cylindrically. It makes sense after reading this, but casting solid tanks in other shapes without seams might also be possible? It’s certainly safer to only go with pressure-vessel shapes.

9. I learned that mechanically, special stainless steel would be a better fit for the SR-71 than titanium, but the process to make and assemble it required clean room standards, which were too much for Skunkworks.

10. Welds failed, and through exhaustive analysis it was found that summer welds failed while winter welds held. The reason was that in the summer the city water used to wash the welds contained chlorine, which weakened the metal. In engineering, everything must be accounted for!!

**MATLAB SCRIPT FOR QUESTION 1**

% Three dynamic pressure corridors in psf

q1=800;

q2=1000;

q3=1200;

%call MATLAB atmosphere script, build atmosphere in 100' increments to

%160,000 feet

[temp,press,rho,Hgeopvector]=atmosphere(0:100:160000,0);

% Mach = v/a

% v = sqrt(2\*q./rho)

% a = sqrt(gamma\*R\*T)

R=1716.55; %ft^2/(sec^2degR)

gamma=1.4;

a = sqrt(gamma\*R.\*temp);

% 1. Using the standard atmosphere model provided, compute conditions

% along constant q trajectories of q=800, 1000, and 1200 psf. Produce

% the following plots:

% a. The flight corridor relating Mach number (x-axis) to

% altitude (y-axis) for each q (on the same plot)

% Build x-axis Mach number, M=v./a

M1 = sqrt(2\*q1./rho)./ a;

M2 = sqrt(2\*q2./rho)./ a;

M3 = sqrt(2\*q3./rho)./ a;

% Build y-axis altitude. Since Mach was built with rho the vector is

% indexed to rho (and temp). So the 5th element of rho corresponds to the

% fifth element of Mach and corresponds to the fifth element of

% Hgeopvector. Make Hgeopvector the y axis

figure(1);

hold on

plot(M1,Hgeopvector)

plot(M2,Hgeopvector)

plot(M3,Hgeopvector)

plot(25.\*ones(size(Hgeopvector)),Hgeopvector,'--')

title('1a.) Mach vs altitude at constant q corridor')

legend('q=800 psf','q=1000psf','q=1200psf','Orbital Velocity')

xlabel('Mach Number')

ylabel('Altitude (ft)')

hold off

% b. Stagnation temperature vs altitude for the three q values noted

% Stagnation temperature is related to static pressure

% Tt0 = stagnation temp; T0=static temp = temp()

% tau = recovery temperature RATIO, is ratio of stagnation to static temp

% tau = Tt0/T0 = (1 + ((gamma-1)/2) \* M^2)

% multiply by T0 to get Tt0

%limit to element 1100 to keep Mach around 10 so graph remains relevant

tempt=temp(1:1100); %freestream static temp

Ht=Hgeopvector(1:1100);

M1t=M1(1:1100);

M2t=M2(1:1100);

M3t=M3(1:1100);

Tt01=tempt.\*(1+((gamma-1)/2).\*(M1t).^2);

Tt02=tempt.\*(1+((gamma-1)/2).\*(M2t).^2);

Tt03=tempt.\*(1+((gamma-1)/2).\*(M3t).^2);

figure(2);

hold on

plot(Tt01,Ht)

plot(Tt02,Ht)

plot(Tt03,Ht)

title('1b.) Stagnation Temperature vs Altitude at constant q corridor.')

legend('q=800 psf','q=1000psf','q=1200psf')

xlabel('Stagnation Temp (Deg R)')

ylabel('altitude (ft)')

hold off

% c. Recovery temperature vs. altitude for the three q values noted

% Plotting recovery temperature vs M for constant q,

% where Tr=T0\*(1 + ((gamma-1)/2)\*(Pr^(1/3))\*M0^2)

% and Pr is Prandtl's Number.

Pr=0.7323; %Looked up sea level value. Since it varies with temp/press,

% but is less than one for air, I'll just use this as a

% constant for purposes of this exercise.

r=Pr^(1/3);

Tr1=tempt.\*(1+r\*((gamma-1)/2).\*(M1t).^2);

Tr2=tempt.\*(1+r\*((gamma-1)/2).\*(M2t).^2);

Tr3=tempt.\*(1+r\*((gamma-1)/2).\*(M3t).^2);

figure(3)

hold on

plot(Tr1,Ht)

plot(Tr2,Ht)

plot(Tr3,Ht)

plot(1680.\*ones(size(Ht)),Ht,'b--o')

plot(3494.\*ones(size(Ht)),Ht,'r--o')

title('1c.) Recovery Temperature vs Altitude at constant q corridor.')

legend('q=800 psf','q=1000psf','q=1200psf','Melting point of Aluminum 1680R','Melting Point of Titanium 3494R')

xlabel('Recovery Temp (Deg R)')

ylabel('altitude (ft)')

hold off

% d. Stagnation Pressure vs. altitude for the three q values noted

% recovery pressure ratio is same as temperature pressure ratio raised to

% power of gamma/(gamma-1)

% Pt0/P0=(1+((gamma-1)/2)\*M^2) ^ (gamma/(gamma-1))

%limit to element 700 to keep Pt0 below 20k psf so graph remains relevant

P0=press(1:700);

M1p=M1(1:700);

M2p=M2(1:700);

M3p=M3(1:700);

Hp=Hgeopvector(1:700);

Pt01 = P0.\* (1 + ((gamma-1)/2).\*M1p.^2).^(gamma/(gamma-1));

Pt02 = P0.\* (1 + ((gamma-1)/2).\*M2p.^2).^(gamma/(gamma-1));

Pt03 = P0.\* (1 + ((gamma-1)/2).\*M3p.^2).^(gamma/(gamma-1));

pi1=Pt01./P0;

pi2=Pt02./P0;

pi3=Pt03./P0;

figure(4);

hold on

plot(Pt01,Hp)

plot(Pt02,Hp)

plot(Pt03,Hp)

title('1d.) Stagnation Pressure vs Altitude at constant q corridor.')

legend('q=800 psf','q=1000psf','q=1200psf')

xlabel('Stagnation Pressure (psf)')

ylabel('altitude (ft)')

hold off

% e. Static pressure vs. altitude for the three q values noted

% Static pressure Ps1, Ps2, Ps3 for the three q values

% Modeled same as recovery temperature, then isentropically related

% Ps1/P0 = (Tr1/T0) ^ ((gamma-1)/gamma)

% Ps1 = P0 \* (Tau r1) ^ ((gamma-1)/gamma)

Taur1=(1+r\*((gamma-1)/2).\*(M1t).^2);

Taur2=(1+r\*((gamma-1)/2).\*(M2t).^2);

Taur3=(1+r\*((gamma-1)/2).\*(M3t).^2);

Ps1=Taur1.^(gamma/(gamma-1));

Ps2=Taur2.^(gamma/(gamma-1));

Ps3=Taur3.^(gamma/(gamma-1));

%limit to element 700 so graph remains visually relevant below 40k'

Psp1=Ps1(1:700);

Psp2=Ps2(1:700);

Psp3=Ps3(1:700);

Hsp=Hgeopvector(1:700);

figure(5)

hold on

plot(Psp1,Hsp)

plot(Psp2,Hsp)

plot(Psp3,Hsp)

title('1e.) Static Pressure vs Altitude at constant q corridor.')

legend('q=800 psf','q=1000psf','q=1200psf')

xlabel('Static Pressure psf')

ylabel('altitude (ft)')

hold off

**MATLAB SCRIPT FOR QUESTION 2**

close all

clear, clc

% List givens

q=1500; %Single dynamic corridor, psf

CPRtjet=10; CPRtfan=30;

FPR=2; BPR=6;

HB=18500; %Delta H\_B for kerosene, for f<<1, h=1=HB

Cp=0.3; %BTU/(lbm \* degF)

Tt4=2600+460; %Turbine Inlet Temp, 2600F convert to Rankine

Tt8=3500+460; %A/B temp or Ramjet Temp, 3500F convert to Rankine

Ttscram=4000+460; %Scramjet burner Temp, 4000F convert to Rankine

Pc=3000; %Rocket Chamber Pressure, psi

g=32.2; %ft/s^2

%call MATLAB atmosphere script, build atmosphere in 100' increments to

%144,200 feet. At that altitude 1500 psf is orbital velocity. No sense

%going any higher.

[temp,press,rho,Hgeopvector]=atmosphere(0:100:144200,0);

% Mach = v/a

% u0 = sqrt(2\*q./rho)

% a = sqrt(gamma\*R\*T)

R=1716.55; %ft^2/(sec^2degR)

gamma=1.4;

a = sqrt(gamma\*R.\*temp);

u0 = sqrt(2\*q./rho);

M0 = u0./a; %Mach indexed to altitude values for constant q

% Build Engine Relationships

%% Freestream Recovery (all engines)

T0=temp; %atm static temp

Tt0=T0.\*(1+((gamma-1)/2).\*M0.^2); %Total Temperature

TAUr=Tt0./T0; %Recovery Temp Ratio

P0=press; %atm static pressure

PIr=(TAUr).^(gamma/(gamma-1)); %Isentropic relationship

Pt0=PIr.\*P0; %Total Pressure

%% Diffuser (all engines)

% For diffuser starting at station 0 and ending ending at turbofan inlet at

% station 19. Without fan,set FPR=1 to 'skip' this station and go to

% station 2 for compressor.

Pt19=ones(size(M0)); % "pre-allocate for speed"

% Mil Std 5008B for diffuser recovery

i=0;

while i<length(M0)

i=i+1;

if M0(i)<1

Pt19(i)=Pt0(i);

elseif M0(i)<5

Pt19(i)=Pt0(i).\*(1-0.075.\*(M0(i)-1)^1.35);

else

Pt19(i)=Pt0(i).\*(800./(M0(i).^4+935));

end

end

PId=Pt19./Pt0; %Diffuser Pressure Ratio

Tt19=Tt0.\*(PId).^((gamma-1)/gamma); %Isentropic Relationship

TAUd=Tt19./Tt0; %Diffuser Temp Ratio

%% Turbofan

PIf=FPR; %Fan Pressure Ratio

Pt2=FPR.\*Pt19; %Total Pressure at fan exit

Tt2=Tt19.\*(FPR)^((gamma-1)/gamma); %Total Temp at fan exit/ comp inlet

TAUf=Tt2./Tt19; %Fan Temp Ratio

%% Compressor (Turbofan)

PIc=CPRtfan; %Compressor Pressure Ratio with fan

Pt3=Pt2.\*CPRtfan; %Total Pressure at compressor exit

TAUc=(CPRtfan)^((gamma-1)/gamma); %Compressor Temp ratio with fan

Tt3=Tt2.\*TAUc; %Total Temp at compressor exit

%% Burner (Turbofan)

PIb=1; Pt4=Pt3; % Ideal burner, no total pressure change

TAUb=Tt4./Tt3; % Non-isentropic temperature relation

%% Turbine (Turbofan)

f=(Cp/HB)\*(Tt4-Tt3);

Tt5 = Tt4 + (Tt2-Tt3)./(1+f) + BPR.\*(Tt19-Tt2)./(1+f);

TAUt=Tt5./Tt4;

PIt=(TAUt).^(gamma/(gamma-1));

Pt5=PIt.\*Pt4;

%% Afterburner

% Not in this turbofan

%% Nozzle

% Ideal nozzle, PIn=TAUn=1, so can use turbine exit relations to simplify

%% Velocity Ratio (Turbofan)

u9u0=sqrt( (TAUb.\* (TAUr.\*TAUd.\*TAUf.\*TAUc.\*TAUt - 1) )./(TAUr-1));

% u9/u0 is velocity ratio in core

u9fu0=sqrt( (TAUr.\*TAUd.\*TAUf-1) ./(TAUr-1));

% u9f/u0 is velocity ratio in fan

u9f=u9fu0.\*u0; %Fan exit velocity

%Force=m\_dot.\*(u9-u0); Thrust is mass flow times velocity diff.

%Force/m\_dot = u9-u0 Specific Thrust of core is delta v

FStfan=(u0./(1+BPR)).\*((u9u0-1)+BPR.\*(u9fu0-1));

% When velocities equal, no thrust! If u9<u0, net drag!

MF = find( FStfan <0, 1); % Finds when specific thrust first <0

Mf = find( f <0, 1); % Finds when fuel flow first becomes <0

% Fuel flow drops to keep turbine in limits

%Stop plot when engine is net drag (MF reached) or no fuel (Mf reached)

Limit= min([MF,Mf]) -1; % -1 to keep positive

ISP\_tfan= (u0./((f).\*g)) .\* ((u9u0-1) + BPR.\*(u9fu0-1));

figure(1)

plot(M0(1:Limit),ISP\_tfan(1:Limit))

title('ISP vs Mach (IDEAL)')

xlabel('Mach'),ylabel('ISP (s)')

hold on

%% Turbojet (overwrite variables, replot on figure 1

% Freestream recovery and diffuser are same; use same variables.

% No fan section

%% Delete Turbofan (Turbojet)

Pt2=Pt19; Tt2=Tt19;

%% Compressor (Turbojet)

PIc=CPRtjet; %Compressor Pressure Ratio with fan

Pt3=Pt2.\*CPRtjet; %Total Pressure at compressor exit

TAUc=(CPRtjet)^((gamma-1)/gamma); %Compressor Temp ratio with fan

Tt3=Tt2.\*TAUc; %Total Temp at compressor exit

%% Burner (Turbojet)

PIb=1; Pt4=Pt3; % Ideal burner, no total pressure change

TAUb=Tt4./Tt3; % Non-isentropic temperature relation

%% Turbine (Turbojet)

f=(Cp/HB)\*(Tt4-Tt3);

Tt5 = Tt4 + (Tt2-Tt3)./(1+f);

TAUt=Tt5./Tt4;

PIt=(TAUt).^(gamma/(gamma-1));

Pt5=PIt.\*Pt4;

%% Afterburner

% Not in this turbojet

%% Nozzle

% Ideal nozzle, PIn=TAUn=1, so can use turbine exit relations to simplify

%% Velocity Ratio (Turbojet)

u9u0=sqrt( (TAUb.\* (TAUr.\*TAUd.\*TAUc.\*TAUt - 1) )./(TAUr-1));

% u9/u0 is velocity ratio in core

FStjet=(u0).\*(u9u0-1);

% When velocities equal, no thrust! If u9<u0, net drag!

MF = find( FStjet <0, 1); % Finds when specific thrust first <0

Mf = find( f <0, 1); % Finds when fuel flow first becomes <0

% Fuel flow drops to keep turbine in limits

%Stop plot when engine is net drag (MF reached) or no fuel (Mf reached)

Limit= min([MF,Mf]) -1; % -1 to keep positive

%still figure(1)

ISP\_tjet= FStjet./(f.\*g);

plot(M0(1:Limit),ISP\_tjet(1:Limit))

%% Turbojet with Afterburner (overwrite variables, replot on figure 1

% All parts through turbine are same between A/B and non-A/B tjet

% Work balance of turbine remains the same, A/B doesn't drive t or c

%% Afterburner

%Starts with turbine exit at station 5, fuel addition at station 7, and

%ends with station 8 at nozzle throat.

%Ratios are therefore station 8 to station 5

Pt8=Pt5; %Constant pressure combustion

PIAB=1;

% For nonAB only one energy addition, T9/T0 = TAUb

% For AB since there's now two energy additions, T9/T0 = (TAUb \* TAUAB)

% Assume Tt8 is constand and is maximum temp for A/B, given above

TAUAB=Tt8./Tt5;

%T9/T0=TAUb.\*TAUAB

%% Fuel Air Ratio for AB

%TAU\_lambda of AB= TauLAB=Tt8./T0;

TauLAB=Tt8./T0;

%From CH5 derivations and balancing enthalpy across AB,

fAB=(Cp.\*T0./HB).\*(TauLAB-TAUr);

%u9/u0 = (M9/M0) \* sqrt(T9/T0)

%In derivation, (M9/M0) came from relating pressure ratios. PIAB=1, so this

%is the same. Only change is addition of TAUAB into numerator.

%For Tjet,

%u9u0=sqrt( (TAUb.\* (TAUr.\*TAUd.\*TAUc.\*TAUt - 1))./(TAUr-1));

%For A/B Tjet,

u9u0=sqrt( (TAUb.\*TAUAB.\* (TAUr.\*TAUd.\*TAUc.\*TAUt - 1))./(TAUr-1));

% u9/u0 is velocity ratio in core

u9=u9u0.\*u0; %Core Exit velocity

FStjetAB=(u0).\*(u9u0-1);

% When velocities equal, no thrust! If u9<u0, net drag!

MF = find( FStjetAB <0, 1); % Finds when specific thrust first <0

Mf = find( fAB <0, 1); % Finds when fuel flow first becomes <0

% Fuel flow drops to keep turbine in limits

%Stop plot when engine is net drag (MF reached) or no fuel (Mf reached)

Limit= min([MF,Mf]) -1; % -1 to keep positive

%still figure(1)

ISP\_tjetAB= FStjetAB./(fAB.\*g);

plot(M0(1:Limit),ISP\_tjetAB(1:Limit))

%% Ramjet

%Ramjet is Ram Air Method Jet, which means ram air itself provides the

%compression. Therefore no compressor, and no turbine to drive the

%compressor. Also, no burner to drive the turbine, so it's just an

%afterburner in a specially-shaped pipe.

%Basically diffuser, afterburner, and nozzle.

%% Delete Machinery

%Assuming ideal conditions, freestream, recovery, and diffuser are the same

%as above. Delete compressor, burner and turbine so that station 5 is the

%same as station 2

Pt3=Pt2; Tt3=Tt2; %Delete compressor

Pt4=Pt3; Tt4=Tt3; %Delete burner that drives turbine

Pt5=Pt4; Tt5=Tt4; %Delete turbine

%% Ramjet burner

Pt8=Pt5; % IDEAL, Constant pressure combustion

%Tt8 is given as max ramjet temp, same as afterburner temp

%with same limitation, essentially a turbojet with no machinery losses, and

%no Tt4 limitation

TAUram=Tt8./Tt5;

%Seeing pattern with other systems, velocity ratio is same as turbojet, but

%only one energy addition in ramjet, and no compressor/turbine ratios

u9u0=sqrt( (TAUram.\* (TAUr.\*TAUd - 1))./(TAUr-1));

FSram=(u0).\*(u9u0-1); %Specific Thrust

fram=(Cp/HB)\*(Tt8-Tt5); % Fuel air ratio across burner section like

% turbojet, but moved to ramjet burner

% When velocities equal, no thrust! If u9<u0, net drag!

MF = find( FSram <0, 1); % Finds when specific thrust first <0

Mf = find( fram <0, 1); % Finds when fuel flow first becomes <0

% Fuel flow drops to keep burner in limits

%Start plot at minimum Ramjet start speed, at M0=1.7

M0ram=find(M0>=1.7,1);

%Stop plot when engine is net drag (MF reached) or no fuel (Mf reached)

Limit= min([MF,Mf]) -1; % -1 to keep positive

%still figure(1)

ISP\_ram= FSram./(fram.\*g);

plot(M0(M0ram:Limit),ISP\_ram(M0ram:Limit))

%% Scramjet

%Scramjet is Supersonic Combustion Ram Air Method Jet, which means ram air

%itself provides the compression. However, the air is still supersonic

%by the time it reaches the burner, and oblique shocks exist inside the

%burner. To accurately model requires Rayleigh flow, and knowledge of Mach

%numbers througout the engine. Basically needs a kinetic energy balance.

% Per Dr. Heister's email response, this will simply be modeled as a

% "faster" ramjet because Mach numbers inside the engine aren't given.

% Highly inaccurate, but it's what we have to go on for now.

%% Scramjet burner

Pt8=Pt5; % IDEAL, Constant pressure combustion

% Absolutely untrue in scramjet, but disregard for HW

%Ttscram is given as max scramjet temp, is higher than A/B or ramjet

TAUscram=Ttscram./Tt5;

fscram=(Cp/HB)\*(Ttscram-Tt5); % Fuel air ratio across scramjet burner

%Tt5 is same for Ramjet & Scramjet, so with

%higher Tt8 in scram, higher fuel burn which

%results in lower Isp at first.

%Seeing pattern with other systems, velocity ratio is same as turbojet, but

%only one energy addition in ramjet, and no compressor/turbine ratios

u9u0=sqrt( (TAUscram.\* (TAUr.\*TAUd - 1))./(TAUr-1));

FSscram=(u0).\*(u9u0-1); %Specific Thrust

% When velocities equal, no thrust! If u9<u0, net drag!

MF = find( FSscram <0, 1); % Finds when specific thrust first <0

Mf = find( fscram <0, 1); % Finds when fuel flow first becomes <0

% Fuel flow drops to keep burner in limits

%Scramjet starts at M0=3.5

M0scram=find(M0>=3.5,1);

%Stop plot when engine is net drag (MF reached) or no fuel (Mf reached)

Limits= min([MF,Mf]) -1; % -1 to keep positive

%still figure(1)

ISP\_scram= FSscram./(fscram.\*g);

plot(M0(M0scram:Limits),ISP\_scram(M0scram:Limits))

%% Rocket

%Run CEA at a number of altitudes to determine Isp of Rocket to Mach...

%One CEA parameter is Pc/Pe, or chamber pressure to exit pressure. I'll

%choose ten exit pressures along the Mach corridor from Mach 1 to Mach 7.

Mhi=find(M0>7,1); %Element where Mach first exceeds 7

Mr=round(linspace(1,Mhi,10)); %Builds evenly spaced element numbers

Pe=ones(size(Mr)); %Pre-allocation

Mrocket=ones(size(Mr)); %Pre-allocation

i=0;

while i<length(Mr)

i=i+1;

Pe(i)=press(Mr(i)); %Convert psf to psi

Mrocket(i)=M0(Mr(i));

end

% Now have evenly space Pe (in psi) indexed to Mach from 1-7.

PcPe=Pc./Pe; % Used to find ISP from CEA website

Ispms=[ 893.4, 1253.9, 1545.4, 1795.9, 2025.4, 2226.5, 2404.8,...

2562.4, 2704.8, 2831.4 ]; %copied from word document

Isp=Ispms./9.8; %converted from m/s to s

plot(Mrocket,Isp)

xlim([1,7])

legend('Turbofan','Turbojet','Afterburning Turbojet',...

'Ramjet (M0max=6.48)','Scramjet (M0max=6.90)','Rocket')